

FOREVER MINUS A DAY? CALCULATING OPTIMAL COPYRIGHT TERM

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ABSTRACT. The optimal term of copyright has been a matter for extensive debate over the last decade. Based on a novel approach we derive an explicit formula which characterises the optimal term as a function of a few key and, most importantly, empirically-estimable parameters. Using existing data on recordings and books we obtain a point estimate of around 15 years for optimal copyright term with a 99% confidence interval extending up to 38 years. This is substantially shorter than any current copyright term and implies that existing terms are too long.

Keywords: Copyright, Copyright Term, Intellectual Property, Optimal Policy

JEL Classification: O31 O34 L10

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1. INTRODUCTION

The optimal term of copyright has been a matter of some importance to policymakers over the last decade. For example, in 1998 the United States passed the Copyright Term Extension Act (CTEA) which extended the length of copyright from life plus 50 to life plus 70 years, applying this change equally to existing and future work (it was during debate on this proposal that it was proposed that copyright last for ‘forever minus a day’).¹ More recently, in the European Union, and especially in the UK, there has been an much debate over whether to extend the term of copyright in sound recordings from its current 50 years.² Given this, the task of calculating optimal copyright term is one with significant importance to current policy-making – as well as being an interesting theoretical and empirical challenge in its own right.

The basic trade-off inherent in copyright is a simple one. On the one hand, increasing copyright yields benefits by stimulating the creation of new works but, on the other hand, it reduces access to existing works (the welfare ‘deadweight’ loss). Choosing the optimal term, that is the length of protection, presents these two countervailing forces particularly starkly. By extending the term of protection the owners of copyrights receive revenue for a little longer. Anticipating this, creators of work which were nearly, but not quite, profitable under the existing term will now produce work, and this work will generate welfare for society both now and in the future. At the same time, the increase in term applies to all works including existing ones – those created under the term of copyright before extension. Extending term on these works prolongs the copyright monopoly and therefore reduces welfare by hindering access to, and reuse of, these works.

It is these two, contrary, effects which will form the main focus of our investigation here. Together they provide plentiful matter for theoretical and empirical efforts. However we should note that we will limit ourselves in at least two important respects. First, much creative endeavour builds upon the past and an extension of term may make it more

¹For example it was referred to by Mary Bono, widow of the musician Sonny Bono, in a congressional speech prior to the enactment of the CTEA: “Actually, Sonny wanted the term of copyright protection to last forever. I am informed by staff that such a change would violate the Constitution. . . . As you know, there is also Jack Valenti’s [President of the Motion Picture Association of America] proposal for the term to last forever less one day. Perhaps the Committee may look at that next Congress.” (CR.144.H9952)

²The last decade or two has also witnessed an unprecedented global spate of extension, driven in no small part by the growing presence of IP clauses in trade negotiations. For example, Australia extended its copyright term from life plus 50 to life plus 70 in 2004.

difficult or costly do so. Were Shakespeare's work still in copyright today it is likely that this would substantially restrict the widespread and beneficial adaptation and reuse that currently occurs. However we make no effort to incorporate this into our analysis despite its undoubted importance (it is simply too intractable from a theoretical and empirical perspective to be usefully addressed at present).³ We will also ignore questions of ex post investment, that is investment by a copyright owner after creation of the work, as well as inefficient exploitation, that is failure by a copyright owner to maximize the value of the work in their possession.⁴

Central to our efforts is a new theoretical framework designed with the primary goal of allowing us to bring to bear the available empirical resources. Our framework starts from that already in the literature (see e.g. Landes and Posner (1989); Watt (2000)),⁵ but extends it in key ways. In particular, following the approach of Pollock (2007), we formally introduce a dynamic, temporal, structure along with the concept of 'cultural decay'. This allows us to be completely clear about the perspective from which to conduct comparative static calculations, in particular those related to optimal term. It also allows us to connect supply and usage via the shared relationship to 'cultural decay' of producer incomes and societal welfare. Together these permit us to derive a single simple equation which defines optimal copyright term as a function of exogenous variables potentially estimable from available data: the discount rate, the rate of 'cultural decay', the supply function for creative work and the associated welfare (and deadweight-loss) associated with new works.

³Some recent theoretical work, e.g. Bessen and Maskin (2006), has shown that the negative effects of IP rights on reuse may be substantial.

⁴See e.g. Landes and Posner (2003) on ex post investment and Brooks (2005) for evidence on inefficient exploitation by rights-owners. We should note that, in our opinion, both of these effects are likely to be relatively limited, and hence we believe their omission, unlike that of 'reuse', is unlikely to have a serious impact on the overall results.

⁵There are, of course, analogies between the optimal patent literature commencing with Nordhaus (1969) and the optimal copyright literature. However the differences are such that the two areas remain largely distinct. Specifically because, crudely, patents are for 'ideas' while copyright is for 'expression' the issues of reuse and breadth while central to patent questions are much less important to copyright ones – similarly while in the patent literature it makes sense to consider several agents 'racing' for a specific innovation this has little meaning in copyright where works are so diverse and no two individuals are likely to produce something so directly substitutable. Conversely reproduction (the making of the 'copy') is a major factor in the analysis of copyright but is essentially irrelevant in the consideration of patents.

Estimating this equation using new and existing data sources we are able to provide one of the first theoretically *and* empirically grounded figures for the optimal length of copyright.⁶

2. A BRIEF NOTE ON COPYRIGHT TERM

The reader should be aware that the term of copyright varies both across jurisdictions and across types of protected subject matter. The right in a recording – as opposed to the underlying composition – is considered a ‘neighbouring right’ and is treated differently from a normal ‘copyright’. In particular, signatories to the Berne convention (and its revisions) must provide for an ‘authorial’ copyright with a minimal term of life plus 50 years, recordings need only be protected for 50 years from the date of publication.

Furthermore, and rather confusingly, works can sometimes be moved from one category to the other as was the case with film in the UK following the implementation of the 1995 EU Directive on ‘Harmonizing the Term of Copyright Protection’ (which ‘harmonised’ copyright term up to life plus 70 years). Prior to this UK law had treated the copyright in the film itself as a neighbouring right and therefore accorded it a 50 year term of protection. Following the implementation of the Directive, the copyright in a film became an ‘authorial’ copyright and subject to a term of protection of life plus 70 years.⁷

3. THE MODEL

In this section we introduce the framework used to derive the main theoretical results. We begin with a very brief exposition of the traditional ‘static’ approach. We then proceed to a more general ‘inter-temporal’ or ‘dynamic’ model in which the fact that works are created and persist over time is taken into account. In full generality the inter-temporal

⁶As Png (2006) notes, there is a lack of empirical work on copyright generally, and existing estimates of optimal term are very sparse. Boldrin and Levine (2005) calibrate a macro-oriented model and derive a figure of 7 years for optimal term in the United States. However this model’s estimates are primarily driven by arguments about market-size and assumptions of past optimality of (much shorter) terms in the United States. (Akerlof et al., 2002) in an examination of the US Copyright Term Extension Act argue, simply on the basis of the discount rate, that a term of life plus seventy years must be too long but do not provide any explicit estimates. By contrast, Liebowitz and Margolis (2005), argue that the current US term of life plus 70 years might not be too long – though again they too do not provide an explicit model or empirical data.

⁷That was not all, as Cornish and Llewelyn (2003, para. 10-45) note, ‘the very considerable investment which goes into major film productions was held to justify a special way of measuring lives. To guard against the consequences of the director’s early death, the longest life among “persons connected with the film” is taken; and these include not only the principal director but the author of the screenplay, the author of the dialogue and the composer of any specifically created film score.’

approach is intractable so we reduce to the evaluation of ‘steady-state’ outcomes. Introducing ‘cultural decay’ we are able to consolidate the set of available works (from all previous periods) into a single figure and obtain a simple expression for (expected) revenue. Combining of this all results in equation (3.10), which gives a simple expression for (per-period) welfare in the steady-state, and can be used to calculate an expression for optimal copyright term in the following section.

Throughout, the strength of copyright, which is here taken to be the term of protection, is represented by the continuous variable S with higher values implying stronger copyright. For the purposes of our analysis copyright may be modelled simply as a monopoly with the term being the period for which that monopoly is granted. Upon expiry the work may be reproduced and reused freely.⁸

3.1. The ‘Static’ Approach. Total welfare, W , is a function of the number of works produced, N , and the term of protection: $W = W(N, S)$. For each work actually produced copyright acts as a monopoly, reducing access and increasing deadweight loss. Thus, the direct effect of longer term on welfare is negative: $W_S < 0$ (subscripts indicate partial derivatives). At the same time welfare will be increasing in the number of works produced: $W_N > 0$.

At the same time an increase in the strength of copyright may increase the (expected) income for the producers of works. This in turn may stimulate the production of works. Thus, the number of works produced is also a function of the level of copyright. We may therefore write $N = N(S)$ and obtained the reduced form form for welfare:⁹

$$W = W(S) = W(N(S), S)$$

Thus, finding the optimal copyright becomes a matter of finding solutions to the classic first order condition:

$$W' = 0 \Leftrightarrow \underbrace{W_N N_S}_{\text{Welfare gain from new works}} = \underbrace{|W_S|}_{\text{Welfare (deadweight) loss on existing works}} \quad (3.1)$$

⁸Of course copyright is subject to various exceptions. However these are not the focus of investigation here and may be safely left to one side. Similarly there are subtleties as regards when the monopoly actually commences (creation versus publication etc). Again these can be safely left to one side.

⁹Note that W and N may also depend on other variables such as the cost of production, the level of demand etc. However, since these are not of concern here we need not explicitly include them.

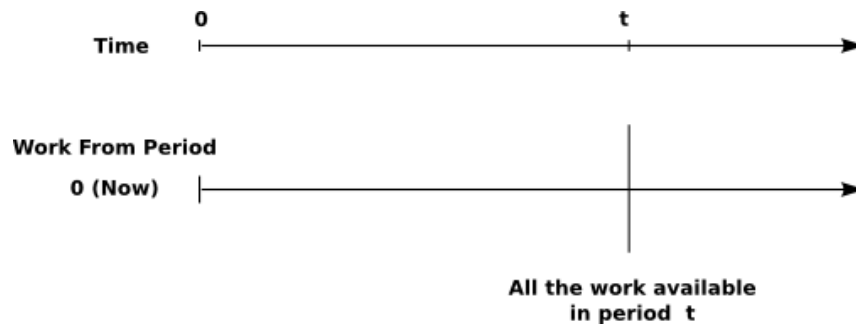


FIGURE 1. The Static Model. Work is produced in only one period ('Now').

3.2. A Dynamic Approach. While attractively simple, the static approach has serious deficiencies when analysing term.¹⁰ Specifically, term necessarily brings in temporal considerations and therefore brings to the fore the durable nature of copyrightable works. In such circumstances the above model, implicitly at least, assumes a single production period ('now/today') with that set of works progressing into the future. This situation is illustrated in Figure 1.

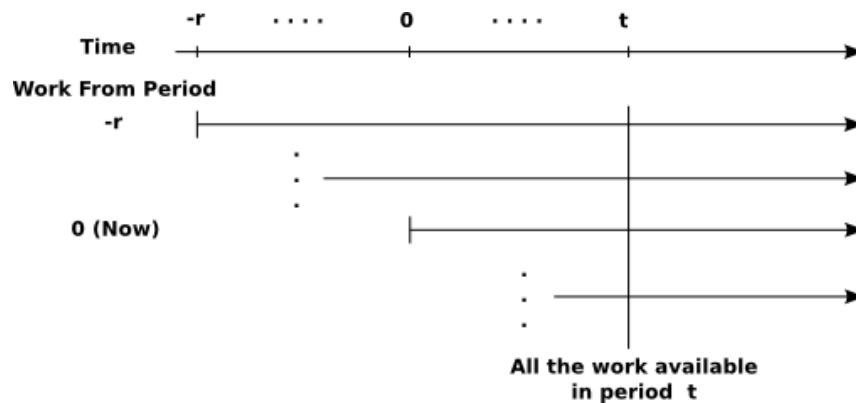


FIGURE 2. The Dynamic Model. Periods are indexed by integers with 'Now' labelled as period 0. Production occurs in every period and work persist through time so at time t there is work available from production in all previous periods.

Presented like this the problem should be obvious: the lack of any production in any period other than the first period (period '0'). Especially when considering a feature such as term it will be important to allow for both the production and persistence of works over time. This alternative approach is illustrated in Figure 2.

¹⁰Difficulties not encountered when analysing other variables, for example the scope of exceptions, or the effect of exogenous variables on the level of protection – see Pollock (2007) for an application of this exact model to some of these questions.

Again, let us label ‘Now’ as period 0. Let periods be indexed by integers t . Within each period there are potential works which can be produced which we index by k_t . Let:¹¹

$$R_{k_t} = \text{(Expected) Revenue PV (in period of production) of work} \quad (3.2)$$

$$C_{k_t} = \text{Fixed Cost of work} \quad (3.3)$$

$$n_t = \text{Number of works produced in period } t \quad (3.4)$$

$$W_t = \text{Welfare in period } t \quad (3.5)$$

Note that revenues should be taken as net of variable costs so that C is solely fixed costs. These variables depend on term, S , and will also interrelate with each other as follows:¹²

$$W_t = W(S, \{n_i\}_{i=0}^t) \quad (3.6)$$

$$R_{k_t} = R_{k_t}(S, \{n_i\}_{i=0}^S) \quad (3.7)$$

$$n_t = n(\{R_{k_t}, C_{k_t}\}_{k=0}^{\infty}) \quad (3.8)$$

Welfare is function of production up to that period and term S (as term determines the welfare we obtain from each work); revenue is function of past and future¹³ production (because of competition) and term; the number of works produced is a function of the (expected) revenues and costs for the set of all works potentially producible in that period. Total welfare evaluated at time T is the sum of discounted welfare (where $d(i)$ is the discount to period i):

$$W_T^{Tot}(S) = \sum_{i=0}^{\infty} d(i)W_{T+i}(S) \quad (3.9)$$

3.2.1. *Reducing to the ‘Steady-State’.* Presented in its full generality this problem is quite daunting. In particular, given the starting conditions (production up to the present), one must determine, for any given term, the full path of present and future production. This is likely to prove an intractable problem except in a very restrictive set of circumstances.

¹¹It is worth noting right at the start the obvious analogies of this problem with standard macroeconomic dynamic models (replace works by capital, welfare by utility of consumption etc). This analogy will be exploited again and again in what follows and is explored in greater depth in the dynamics section of Pollock (2007).

¹²Clearly there are many other exogenous variables that would affect all of these – the size of demand, technology etc – but these are not our focus and for simplicity of notation they are not formally included.

¹³Future outcomes would of course be unknown but could be taken as equal to their expected value.

Furthermore, with these interrelations of production in the past, present and future, production dynamics could be quite complex and optimal protection would likely vary over time giving rise to the question of what was meant by *the* optimal term.¹⁴

The classic solution to this problem is presented by the analogy with similar macroeconomic questions: the calculation of comparative statics, such as the evaluation of optimal copyright term, can be done at the steady-state, in which production is constant at the steady-state level.¹⁵

This greatly simplifies matters as one no longer need consider a whole series of different levels of production n_t in each period. Instead one has a single value for the level of production, n , which is a function of a single set of revenues and costs: R_k, C_k . Steady-state welfare is then constant:

$$W = W(S, \{n\}_{-\infty}^0)$$

The next step is to simplify the welfare function. As defined, welfare is a function of production in every previous period. This reflects two distinct facts. First, works are durable – we still enjoy *A Midsummer Night’s Dream* or *Oliver Twist* despite being created centuries or decades ago. Second, the works from the past are not necessarily directly comparable, at least en masse, with those of today: while some older works continue to be highly valued today the great majority are utterly forgotten. However, it would be

¹⁴For example as the stock of works increased optimal protection would fall. This was an issue explored, together with this general dynamic model, in more depth in Pollock (2007).

¹⁵Strictly, we are not guaranteed that an equilibrium is of this form – we could have production cycles or even chaotic dynamics. However, we leave such complex possibilities aside here. Of course, we must be careful, as in the macroeconomic case, about comparison of steady state outcomes without consideration of the transition between the two states. Specifically, suppose that current term is S and that we determine that optimal term is T . Though we know that *in the steady state* T is better than S , in order to determine the merits of a switch to T from S requires consideration of welfare over the transition path when we switch from T to S . This is an important issue – and one considered at some length in Pollock (2007), but here we will ignore it for three reasons.

First, the transition issue is a secondary one: calculating optimal term from an “a priori”, steady-state, perspective must come before we get into the question of the transition path. Second, the theoretical and empirical difficulties are already substantial in the steady-state analysis and any explicit calculations for the transition are likely to present serious, perhaps even insurmountable, difficulties. Third, and more importantly, when current term is higher than optimal term – as is found to be the case in this paper – taking account of welfare over the transition is likely to only reinforce the basic welfare result based on comparison of steady-states. (Why? When optimal term is less than current term, current production levels are above those in the steady-state optimum. This, in turn, implies that welfare over the transition is likely to be even higher than at the optimal steady-state level because the stock of works is higher than at the final steady-state level (T) due to the backlog of works produced under previous higher copyright regime, while access has increased immediately – assuming the term is changed retrospectively as well as prospectively).

very useful to be able to combine these many different periods of production into a single value. This we do by introducing the concepts of cultural decay and the stocks of works.

3.2.2. *Cultural Decay and Steady-State Welfare.* In a given period, a user of a work will not care directly about its ‘vintage’ – that is, when it was produced. This suggests that one might be able to aggregate the welfare from works from different periods into a single value in a simple (linear) way. As just discussed, the value of a set of works from a previous era tend to be less than an equivalent number of today’s due to the effects of cultural depreciation and obsolescence. It is therefore natural to create for the welfare (and revenue) from cultural works a ‘cultural decay’ (or depreciation) rate similar to the depreciation rate for traditional capital.

Formally, we define $b(i)$, the ‘rate of cultural decay’ after i time periods ($b(0) = 1$) as follows. Suppose we have n works produced today. Let their total (i.e. ignoring deadweight losses from copyright monopoly) welfare generated today be $Y(n)$ – we shall refer to this as ‘per-period’ (total/potential) welfare.¹⁶ Then n works which are i periods old yield total welfare (today) of $b(i)Y(n)$ – i.e. the i period cultural decay rate times their original value.

Now the value of a work or set of works (that is, the welfare generated) can be divided into two parts: a) the total welfare b) the deadweight-loss (under copyright) which only becomes available once in the public domain. Let $Z(n)$ be ‘per-period’ deadweight loss on n works, that is the deadweight-loss today on n current works. Then, for works in copyright, the welfare actually generated from a set of n works is $Y(n) - Z(n)$. Using this, and defining $B(T) = \sum_{i=0}^T b(i)$, steady-state welfare becomes:

$$W = W(S, \{n\}_{-\infty}^0) = \sum_{i=0}^{\infty} b(i)Y(n) - \sum_{i=0}^S b(i)Z(n) = B(\infty)Y(n) - B(S)Z(n) \quad (3.10)$$

In words: welfare is the sum of ‘total’ per-period welfare from each of the periods of production (from today into the distant past) minus deadweight loss on works still in

¹⁶The ‘per-period’ emphasizes this is welfare generated by these works in a single period – not over their lifetime.

copyright (those from the previous S periods) with the welfare of works from previous periods being converted into current levels via cultural decay factors.¹⁷

There are also some important subtleties in this formulation. In particular, observe that we are implicitly assuming: a) the separability of welfare at the work level (into welfare under copyright and deadweight loss) can be carried across to the set of works from a given period; b) total welfare (which is a function of all works, past and present) is separable into a sum of per period welfare weighted by cultural decay factors. The first assumption is simple enough that we do think it warrants further discussion here. The second, however, does warrant such further discussion.

Specifically, consider the impact of a marginal increase in per-period production when its level is initially n . This results in some extra work being produced per period dn say. This in turn results in a total extra set of work today of $dnB(\infty)$. Thus, $Y(n)$ measures the marginal value of $dnB(\infty)$ new works when there are currently $N = nB(\infty)$ works available (not n as one might think).

This is important because when thinking about cultural work (or any other durable item) it is necessary to distinguish between two distinct causes of diminishing returns. First, is the ‘stand-alone’ effect: some works are more valuable than others and the more valuable ones are likely to get produced first (this is the classic cause of diminishing returns).

Second, is the ‘interaction’ or ‘collective’ effect: with more and more works available there is less need for new work, *independent* of the quality of that new work – once we’ve had the Sixties even if subsequent decades deliver equally good rock and pop music it is necessarily less valuable because there is already a large stock of good work in existence. Formally, if one has $y(n) = Y'(n)$ as the marginal value of new work one could think of it being multiplicatively separable $y(n) = f(n)g(nB(\infty))$, where $f(n)$ measures standalone

¹⁷An alternative approach would be to express W as the sum of welfare from individual works with additional separability across time:

$$W = \sum_{j=0}^{\infty} b(j) \left(\sum_{i=0}^n y(i) - z(i) \right)$$

With $y(i)$ being potential welfare from work i and z deadweight loss. The difficulty with this approach is that there is interaction between works – both within periods and, potentially, across time (see discussion in text below). Thus, rather than proceed at the level of individual works we retain (partially) aggregate functions Y, Z which implicitly include this ‘interaction’.

marginal value in per period production,¹⁸ and $g(nB(\infty))$ takes account of the ‘interaction’ effect.¹⁹

3.2.3. Revenue and Cultural Decay. Lastly, we calculate present value revenues for works by applying the cultural decay rates to revenue in the same way they were applied to welfare.

Specifically, let revenue (under copyright) on the k th work in the period t after its creation be given by $r_k(t)$. Upon expiry of copyright, revenues (net of variable costs) are zero – i.e. $r_k(t) = 0$ for t greater than the term of protection, S . R_k , as already defined, is the PV of all revenue accruing to the work over its lifetime. To augment this, define $R_k(T)$ to be the present value of all revenue up to period T . Let $d(i)$ be, as above, the discount factor up to time i , then:

$$R_k(T) = \sum_{t=0}^{t=T} d(t)r_k(t) \quad (3.11)$$

The last step is to link use (and welfare) to production and revenue. In particular, the cultural decay rates applied to past works can naturally be applied to the revenues of new works as they age into the future – a work of age t from the perspective today is also of age t from the perspective of its original creator. Thus, it is natural to specify that revenue in period t equals revenue in period 0 multiplied by the cultural decay rate for t periods (up until the expiry of copyright at which point $r_k(t) = 0$):

$$r_k(t) = b(t)r_k(0) \quad (3.12)$$

Thus total (PV) revenue to time T (for T less than the term of copyright S) becomes:

$$R_k(T) = \sum_{t=0}^{t=T} d(t)b(t)r_k(0) \quad (3.13)$$

Note that implicit in this formulation are two significant assumptions. First, works experience a common rate of cultural decay – that is, the cultural decay rate does not vary across works. This is perhaps more reasonable than it may at first seem in that

¹⁸It is necessarily required by our steady-state analysis that in each new period there is a new set of potential works available identical in the distribution of their ‘stand-alone’ value to the works of other periods.

¹⁹An natural model to apply here is that of monopolistic competition. In that setup $g(N)$ ($N = nB(\infty)$) takes the form N^α for some $\alpha \in (-1, 0)$.

revenue calculations are forward-looking and hence strictly we should be using the *expected* cultural decay rate. Ex-ante there might be no reason to distinguish between the level of cultural decay across works and so all creators would use the same set of expected decay rates.²⁰

Second, the same cultural decay rate used for welfare can be applied to the revenue of works. Here again the fact that decay rates for revenue are the expected decay rates plays a role. With uncertainty about these rates it would be natural for creators to use as their estimate the *average* decay rate of works up until now, i.e. the existing welfare decay rate.

4. THEORETICAL RESULTS

Since we evaluate welfare at the steady-state, with production per period at a constant steady-state level, welfare per period is also constant at this steady-state level. Therefore in what follows one can focus on welfare per period, W , rather than on total welfare (equal to the discounted sum of welfare in all future periods: $W^{Tot} = \sum_{i=0}^{\infty} d(i)W$). Before stating the main result we summarize the existing notation:

$d(t)$	=	Discount factor to time t
$b(t)$	=	Cultural decay to period t
$D(T)$	=	$\sum_{i=0}^T d(t)$, $B(T) = \sum_{i=0}^T b(t)$, $DB(T) = \sum_{i=0}^T d(t)b(t)$
$n(S)$	=	Steady-state per period production (PPP)
$Y(n)$	=	Welfare ‘per period’ as a function of PPP
$y(n)$	=	Marginal welfare ‘per period’ when PPP is n
$Z(n)$	=	Deadweight-loss (under copyright) ‘per period’
$z(n)$	=	Marginal d/w-loss ‘per period’ as function of PPP
$s(n)$	=	Elasticity of supply with respect to revenue
$\theta(n)$	=	Ratio of avg. d/w loss to welfare from new works = $\frac{Z(n)/n}{s(n)y(n)}$

²⁰In addition, those funding creative production might hold a *portfolio* of works. If so this would clearly move their expected decay rate closer to the average decay rate.

Proposition 1. *Optimal copyright term S is a solution of $\Delta = 0$ where Δ is the ‘determinant’ given by:*

$$\Delta = \frac{d(S)}{DB(S)} \left(B(\infty) - B(S) \frac{z(n)}{y(n)} \right) - \theta(n) \quad (4.1)$$

At least one such solution exists and under mild conditions it is unique.

Proof. See appendix. □

The first term (that multiplied by the discount factor) relates to the proportional increase in revenue as a result of an increase in copyright term, while the second is, crudely, a ratio of the increase in deadweight loss to the increase in welfare from new works.

The most important feature here is that only the first term has any dependence on the discount factor. This reflects the fact the discount factor directly affects the production of new work – extra revenue from an increase in term will be discounted by $d(S)$ – while it has no direct impact on deadweight loss.²¹ This contrasts directly with the static model as in that setup there is no ‘past’ production and hence discounting affects the deadweight loss and production term approximately equally.

Specifically the first term is multiplied by: $\frac{d(S)}{DB(S)}$. The numerator here is the discount factor to time S . Its presence reflects the fact that producers of new work care about the extra revenue from new works S periods in the future. The denominator is the sum of unit payments up to time S discounted by the discount factor and cultural decay. It corresponds, crudely, to the amount of revenue a producer already has and its presence reflects the fact that the importance of new revenue is determined relative to how much a producer already has.

Another important feature, particularly significant from an empirical point of view, is that the terms related to the demand curve – $y(n), z(n)$ – occur in ratios. Recall that $y(n)$ is the (total) welfare from the n th work while $z(n)$ is the deadweight loss. Obtaining these figures is usually very hard because we have little information about the full demand curve for a work. Furthermore, it is clear that these figures will vary greatly across works (i.e. as n varies) – some books sell a million copies (high y) while some sell almost none (very low y). However, the ratio of deadweight loss to total welfare, $z(n)/y(n)$ is likely

²¹By contrast cultural decay affects *both* dead-weight loss and the production of new work (via its impact on producer revenues), and does so approximately symmetrically (with a term of S the marginal public domain work has decay $b(S)$ and the marginal revenue for producers has decay $b(S)$)

to vary much, much less. This is because total welfare and deadweight loss are likely to be strongly correlated – in fact, it would not be unreasonable to assume that this ratio is approximately constant across a wide set of works, even though the value of the works themselves varied greatly. Thus, by having a formula expressed in terms of these ratios, estimating optimal term becomes both easier and less prone to error.²²

5. ESTIMATION

The previous section derived a relatively simple expression which characterised the optimal copyright term in terms of the zeroes of the determinant, Δ . The next task is to obtain estimates for that equation's various component variables (b, d, θ etc). This section will go through each of the variables in turn, starting with the simplest to estimate (the discount rate) and progressing to the hardest (θ).

At this juncture, it is important to point out that we will only obtain a single set of estimates and hence a single value (or range) for optimal copyright term. As is clear upon a cursory examination of copyright's usage (see Section 2), copyright applies to a variety of domains, and the term of protection often varies according to the domain.²³ These distinctions fit naturally within our framework. For example, while it is natural to take the discount factor d as constant across all areas, it might well be the case that the rate of cultural decay varies – perhaps being higher for music (especially popular music) than it was for fiction books. Similarly, it is plausible that other variables such as the elasticity of supply, or the rate of diminishing returns to new work vary across media and activity.²⁴

In a perfect world, one would obtain estimates for the variables each distinct area and then calculate an area-specific copyright term.²⁵ However, as will be seen below, given the data presently available, it is hard enough to obtain one estimate let alone estimates for a variety of different areas. Thus, while we would highlight this 'customization' of the

²²There is one major loss: we will not be able to obtain any quantification of welfare changes using the formula. That is, we will not be able to say something such as: 'a reduction in copyright term will yield benefits of X in monetary terms.' Instead, we will be confined to statements of the form: 'a term of Y is too long'.

²³In fact, as discussed previously, a single work can have multiple copyrights (or a copyright and a 'neighbouring right'). For example a recording will have an 'authorial' copyright in the composition and a 'recording' right in the recording itself and the terms of these two rights are usually different (the existence of two distinct right fits well with the fact that the composer and the performer may well be different, especially in classical music).

²⁴These variables may also vary across countries and cultures.

²⁵Much as one might want to calculate technology or industry specific patent terms.

empirical estimates as an important area for future work, in the present paper we will focus on providing a single estimate, buttressed by a variety of robustness checks that mean that many of the results can be applied across different areas of activity even in the face of the undoubted variation in some of the underlying parameter across those areas.²⁶

5.1. The Discount Factor. We assume a standard geometric/exponential form for the discount function. The discount rate should be that relevant for those *producing works* – not the general societal discount rate – and as such should be risk-adjusted. Given these considerations a reasonable range is a discount rate in the range 4-9%.²⁷ For example, CIPIL (2006) in considering a similar issue report that: Akerlof et al. (2002) use a real discount rate of 7%, Liebowitz in his submission to the Gowers review on behalf of the IFPI (International Federation of the Phonographic Industry) uses a figure of 5%, while PwC’s report to the same review on behalf of the BPI (British Phonographic Industry) use the figure of 9%. Where we need to use a single value we will by default use a rate of 6% (corresponding to a discount factor of 0.943).

5.2. The Rate of Cultural Decay. We assume an exponential form for the cultural decay so that $b(i) = b^i$ with b the cultural decay factor.²⁸ A plausible range for the cultural decay rate is 2-9% and by default we will use 5% (corresponding to a factor of 0.952). Since values for these variables are less well-established than those for the discount rate the evidence on which they are based merits discussion.

The prime source is CIPIL (2006), which reports estimates made by PwC based on data provided by the British music industry which indicate decay rates in the region of 3-10%.

²⁶It is also worth noting that while, as discussed, there is some variation in the term of copyright and its related rights, for example, the basic ‘authorial’ term is constant in most jurisdictions across the major media – books, music, films, even software. The previous discussion indicates why it is unlikely that this is optimal, nevertheless it perhaps reflects the same lack of any concrete data on which to base variations in term that underly our decision to focus on single ‘optimal term’.

²⁷Values at the lower end of this range must be considered fairly generous given the uncertainty, ex-ante, in the value of works. Of course, many of the investors in the production of copyrightable work are large widely diversified multinational enterprises. Nevertheless, even for them risk-adjusted discount rates would be at the upper end. For example, PwC’s figure was specifically generated for the case of large publicly-traded music companies.

²⁸It is likely that an exponential distribution is not a perfect fit for the cultural decay rate. In general, it appears that the rate of decay is sharper than an exponential for young works but flatter than an exponential for old works. This suggests that hyperbolic cultural decay might be a better model (just as hyperbolic discounting may be more accurate than exponential discounting for income). However, an exponential form appears to be a reasonable approximation and it is substantially more tractable. Thus we retain it here rather than using the more complex hyperbolic approach (just as an exponential form is regularly used for time discounting for analogous reasons).

As these come from the music industry itself, albeit indirectly, these have substantial authority. To check these we have performed our own calculations using data on the UK music and book industry and obtain estimates for the rate of decay that are similar (in the case of music) or even higher (in the case of books).

Evidence from elsewhere includes the Congressional Research Service report prepared in relation to the CTEA (Rappaport, 1998). This estimates projected revenue from works whose copyright was soon to expire (so works from the 1920s to the 1940s). Rappaport estimates (p.6) that only 1% of books ever had their copyright renewed and of those that had their copyright renewed during 1951 to 1970 around 11.9% were still in print in the late 1990s. The annual royalty value of books go from \$46 million (books from 1922-1926) to \$74 million (books from 1937-1941). Turning to music, Rappaport focuses on songs (early recordings themselves have little value because of improvements in technology) and finds that 11.3% of the sample is still available in 1995. Annual royalty income rises from \$3.4 million for works from 1922-1926 to \$15.2 million for works from 1938-1941.

These figures correspond, in turn, to cultural decay rates of 3.2% and 10.5% respectively. However these are far from perfect estimates since we only have two time points. Furthermore these time points correspond to different ‘cohorts’ of work – which makes it difficult to disentangle decay effects from cohort effects, and both these cohorts are of fairly old works – which, as explained in a previous footnote means that the decay rate is likely to be underestimated. One might also want to be cautious about extrapolating to the behaviour of current and future creative output from data of such elderly vintage.²⁹

Liebowitz and Margolis (2005) argues that overall decay rates may be misleading and presents evidence that books that are popular upon release as measured by being best-sellers survive well (for example the table on p. 455 indicates that of the 91 bestsellers in their sample from the 1920s 54% are still in print 58 years later compared to only 33% of non-bestsellers).

²⁹The issue of technological change is clearly an important one here: one might argue that with improvements in technology, both in production but also in distribution and discovery, the decay rate will fall in future. For example, it has been argued recently that technologies such as the Internet have made it easier to discover and access more obscure works leading to the growing importance of the ‘long-tail’ and a flattening of the distribution of sales (traditionally sales for most types of copyrightable goods have been dominated by a top 10-20% of works. The ‘long-tail’ then refers to the tail of this sales distribution). Here we do not explicitly consider the impact of technological change but we note that an earlier paper (Pollock, 2007) dealt specifically with this issue.

However it is not clear how one should interpret this sort of evidence. Simple ‘in-print’ status of a book only places a lower-bound on sales (furthermore a lower bound that is dropping with advances in technology) and does not allow us to compare the sales of a book today compared to when it was first released. More fundamentally, much heterogeneity is eliminated by the aggregation of copyrights into portfolios by the investors in creative work such as publishers, music labels and movie studios. In this case returns will tend to the average. Furthermore, were such aggregation not to occur it would require a substantial increase in the discount rate to take account of the increased uncertainty due to the reduction in diversification of the portfolio.³⁰

5.3. Deadweight-Loss, Welfare per work and $\theta(n)$. Our preference would be to estimate all of these values directly from empirical data. However, this is a daunting task given currently available datasets as it requires us to determine: the full demand system for copyright goods *and* the supply function for creative work. Because this task presents such insurmountable difficulties given present data availability we instead take a ‘reduced-form’ approach where we supply particular functional forms for the various quantities of interest (the average deadweight loss, marginal welfare etc). Where possible we calibrate these using existing data and we also perform various checks to ensure these results are reasonably robust. We begin by making the following assumptions:

- (1) The elasticity of production with respect to revenue, $s(n)$, is constant (at least in the region of current output levels), and equal to s .
- (2) The ratio of deadweight-loss to welfare on any given work is constant. This constant will be termed α .
- (3) The ratio of marginal welfare, $y(j)$, to marginal sales is constant. That is welfare follows the same trend as sales. This constant will be termed β .

³⁰In these circumstances the issue of serial correlation would also become important. With high serial correlation – old successful works are those that were successful when young (and vice versa) – the revenue when one extends term goes primarily to the owners works which have already generated substantial revenue (think here of a group like The Beatles). If one makes the standard assumption of diminishing marginal returns to creative output with respect to revenue, then serial correlation implies a very low elasticity of supply with respect to revenue – the revenue from extending term goes to those whose incomes are already high and therefore from whom little extra ‘creation’ can be expected when their incomes increase.

Assumption 1: little if anything is known about how the elasticity of supply with respect to revenue varies with the number of works produced. Furthermore we already allow changes in welfare per work so it is not that restrictive to take elasticity as constant.

Assumption 2: this assumption is questionable as one might expect that deadweight losses relative to welfare (under copyright) increase as the welfare (and revenue) from a work decline.³¹ If this were so then this assumption would be incorrect and would result in an underestimate of the costs of copyright – and hence an overestimate of optimal copyright term. Nevertheless, we shall make this assumption for two reasons. First, it is difficult to derive estimates of this ratio from existing data. Second, as we shall see below, even with it (and the associated upward bias) we find that optimal term is well below the copyright terms found in the real world.

Assumption 3: this requires that the ratio welfare (under copyright) arising from a new work to the sales of that work does not vary over works. Again this is almost certainly not an accurate description of reality but as a first order approximation we believe it is not that bad. Furthermore, this assumption is crucial for our empirical strategy since it is relatively easy to obtain sales data compared to welfare data (which requires information on large segments of the demand curve).

Now, to proceed with the analysis. First recall that $Y(j)$ is total welfare per period and that that y is marginal welfare: $y(j) = Y'(j)$. Define $Q(j)$ as total sales and $q(j) = Q'(j)$ as marginal sales (i.e. sales from the j th work). What form does $Q(j)$ take? We shall assume it takes a ‘power-law’ form:

$$Q(j) = Aj^\gamma$$

This functional form appears to represent a reasonably good fit for sales of cultural goods and is frequently used in the literature.³²

³¹For example, this would be the case if there was some fixed lower bound to transaction costs.

³²See e.g. Goolsbee and Chevalier (2002); Ghose et al. (2004); Deschatres and Sornette (2004). It should be noted though that evidence obtained by the authors on books in the UK suggest that sales actually drop more sharply than a power-law would suggest (i.e. has a thinner ‘tail’). Optimal term will be decreasing in the sharpness of the decline – a sharper decline corresponds to more sharply diminishing returns to new work. So, if correct, this would imply that our estimates of optimal terms obtained below are *over-estimates*. Given that these estimates are already well below the current observed level in order to be ‘conservative’, as well as for the sake of analytical convenience, we have proceeded on the basis of a power-law relationship.

Lemma 2. $\theta(n)$ has the following simple form:

$$\theta(n) = \frac{\alpha}{s\gamma}$$

Proof. Recall that $\theta(n) = \frac{Z(n)/n}{sy(n)}$. Now $y(n) = \beta q(n)$, $z(n) = \alpha y(n)$ so average deadweight loss, $Z(n)/n$ equals $\alpha \frac{\beta Q(n)}{n}$. Hence:

$$\theta(n) = \frac{\alpha \beta Q(n)}{s n \beta q(n)} = \frac{\alpha}{s\gamma}$$

□

Thus, one very convenient aspect of using a ‘power-law’ form is that $\theta(n)$ is not a function of n – it is ‘scale-free’. In this case calculations of optimal copyright term do not depend on, n , the production function for works but only on α , γ and s .

5.3.1. *Exponent for Sales Power Law: γ .* Ghose et al. (2004) list a whole range of estimates for $\gamma - 1$ (all derived from Amazon) ranging from -0.834 to -0.952 with the best estimate being -0.871. These imply γ in the range 0.048 to 0.166 with best estimate at 0.129. We shall proceed using this estimate of 0.129.

5.3.2. *Elasticity of supply with respect to revenue: s .* There is very little data which would allow us to estimate the elasticity of supply with respect to revenue. Landes and Posner (2003) who point out that there is no discernible impact on output of work from the US 1976 extension of term. Hui and Png (2002) find a similar result when looking at movies and the CTEA in the US though more recent work with a cross-country dataset, Png and hong Wang (2007), does find an impact. However, recall that $s(n)$ is the elasticity of supply with respect to revenue of the *marginal* work – i.e. it is the percentage increase in total production from a 1% increase in the revenue of the marginal work (that which is currently nearest to being profitable). As such this elasticity should not be very big.

Consider, for example, a simple model in which (fixed) costs are constant across works at some level f . Revenues meanwhile have some distribution $g(j)$ (as a function of their index j). Without loss of generality we may order revenues by size so that g is non-increasing. Then n is defined by the solution of $g(n) = f$. Take a simple case, corresponding to the sales distribution discussed in the text in which $g = j^{-\eta}$. Suppose initially one has a solution n_0 . Now suppose that revenues all increase by some percentage p : $g \rightarrow g' =$

$(1+p)g$. Then the new n_1 , n_1 solves $(1+p)n_1^{-\eta} = f = n_0^{-\eta}$. Hence $n_1 = (1+p)^{1/\eta}n_0$ and so:

$$s(n) = \lim_{p \rightarrow 0} \frac{n_1 - n_0}{n_0} / \frac{R' - R}{R} = \lim_{p \rightarrow 0} \frac{(1+p)^{1/\eta} - 1}{p} = \frac{1}{\eta}$$

Now, $\eta = -(\gamma - 1)$. Taking the point estimate from the previous section for γ this gives $\eta = 0.871 \Rightarrow s(n) = 1.14$. As discussed earlier it appears likely that a power law approach over-estimates the fatness of the tails of the revenue distribution in which cases these elasticities are too high. Given this, we feel a reasonable parameter range for s would be $[0.5, 1.5]$ with an average value of 1.0.

5.3.3. Ratio of deadweight loss to total (possible) welfare: α . Estimating α is also difficult because of the paucity of data which would permit estimation of off-equilibrium points on the demand curve. However the available evidence though scanty suggests that the ratio could be quite large. For example, Rob and Waldfogel (2006) investigate file-sharing among college students and estimate an implicit value for deadweight-loss of around 36% of total sales. Converting this to welfare ratio requires some assumption about the ratio of welfare to sales. A linear demand structure (with zero marginal costs) would give a deadweight loss to sales ratio of 50% and deadweight loss to welfare ratio of a quarter. Increasing marginal costs would reduce the ratio to sales but keep the ratio to welfare constant at a quarter. Being more conservative, assuming producer surplus were around 50% of sales and consumer surplus to be two to five times would give a value for α between 0.24 and 0.12. Other papers, such as Le Guel and Rochelandet (2005); Ghose et al. (2004), while not providing sufficient data to estimate deadweight loss, do suggest it is reasonably substantial. Thus, we feel a plausible, and reasonably conservative, range for α would be from $[0.05, 0.20]$, that is deadweight loss per work is, on average, from a twentieth to a fifth of the total welfare available from a work. When required to use a single value we will use the halfway point of this range 0.125.

5.4. Optimal Copyright Term: An Estimate. Combining estimates of the ratio of deadweight losses to available surplus (α) and the rate of diminishing returns (γ) with those provided above for cultural decay (b) and the discount factor (d) we will obtain point estimates for optimal copyright term. Taking parameter values in the mid-point of their ranges: gives $\alpha = 0.125, s = 1.0, \gamma = 0.129$ then $\theta \approx 0.969$. With our default

Cultural Decay Rate (%)	Discount Rate (%)	α	Optimal Term
2	4	0.05	51.8
3.5	5	0.07	30.7
5	6	0.1	18.5
6.5	7	0.15	10.6
8	8	0.2	6.5

TABLE 1. Optimal Term Under Various Scenarios. α is the proportion of available surplus that is deadweight loss surplus. s (the elasticity of supply) is set to 1 and γ (sales curve exponent) to 0.129.

discount rate of 6% and cultural decay of 5% this implies an **optimal copyright term of around fifteen and a half years**.

This, of course, is a single point estimate based on taking parameters at the mid-point of their range. Given the uncertainty over the values of some of the variables it is important to derive optimal copyright term under a variety of scenarios to check the robustness of these results. Table 1 presents optimal term under a range of possible parameter values including those at the extreme of the ranges suggested above.

With variables at the very lowest end of the spectrum (the first row) optimal term comes out at 52 years which is substantially shorter than authorial copyright term in almost all jurisdictions and roughly equal to the 50 years frequently afforded to neighbouring rights (such as those in recordings). However as we move to scenarios with higher levels for the listed parameters optimal term drops sharply. For example, with middle-of-the-range values: cultural decay at 3.5%, the discount rate at 5% and the ratio of deadweight loss to available surplus at 7% we already have an optimum term of just 31 years which is substantially below all existing copyright terms. Moving to higher end of parameters listed presented here, with deadweight losses at 20% of available surplus (recall that a linear demand curve corresponds to a 25% ratio), cultural decay at 8% and the discount rate at 8%, optimal term is around six and a half years.

We can also plot a probability density function under the assumed variable ranges. This has the advantage that it incorporates the interrelations of the various variables – by contrast, Table 1, by nature of its form, implicitly gives the inaccurate impression that each of the outcomes listed is equally likely. We present the distribution function in Figure 3. As this shows, the mode of the distribution is just under 20 years. From the underlying cumulative distribution function we can calculate percentiles. The 25th

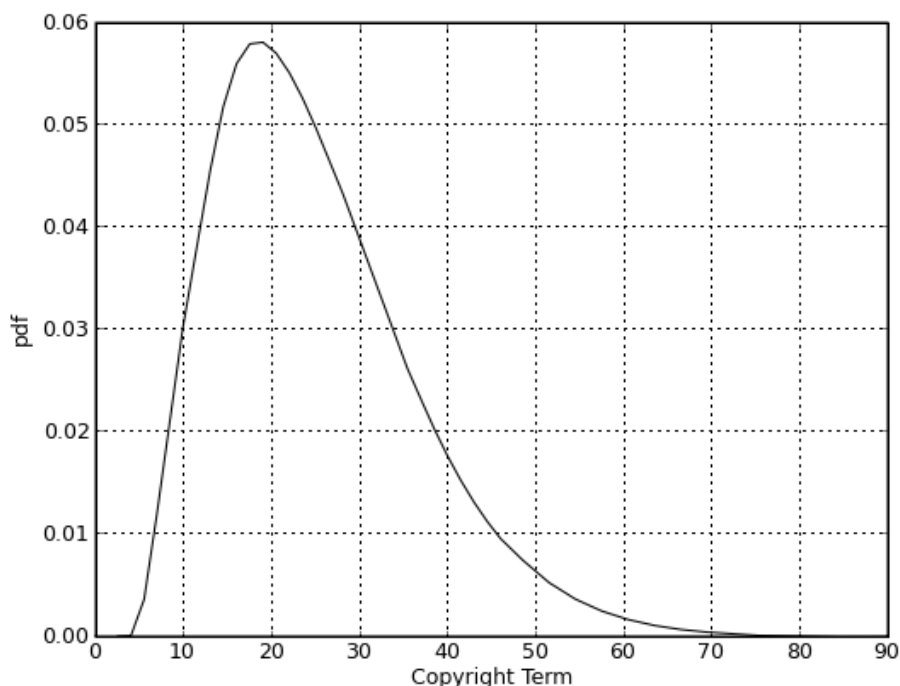


FIGURE 3. Probability distribution of optimal term given the parameter range set out above (with the exception that γ takes a single value of 0.129).

percentile is 11 years, the 50th (the median) at 15 years, the 75th at 21 years and the 95th percentile at 31 years, the 99th percentile at 38 years and the 99.9th percentile at 47 years. This would suggest, that at least under the parameter ranges used here, one can be extremely confident that copyright term should be 50 years or less – and it is highly likely that optimal term should be under 30 years (95th percentile).

5.5. Further Robustness Checks: An Inverse Approach. An alternative approach to estimating underlying parameters and using that to find the optimal term is to look at the inverse problem of calculating the ‘break-even’ value for a particular variable for a given copyright term. The ‘break-even’ value is the level of that variable for which that term is optimal. Here we will focus on α , the ratio of deadweight loss to total available surplus on a work – so if the actual value α is higher than this break-even level then term is too long and if actual α is below it then term is too short. This provides a useful robustness: derive the break-even α corresponding to the copyright term currently in existence and then compare this value to whatever is a plausible range for α . If the value is outside this range one can be reasonably certain that current copyright term is too long.

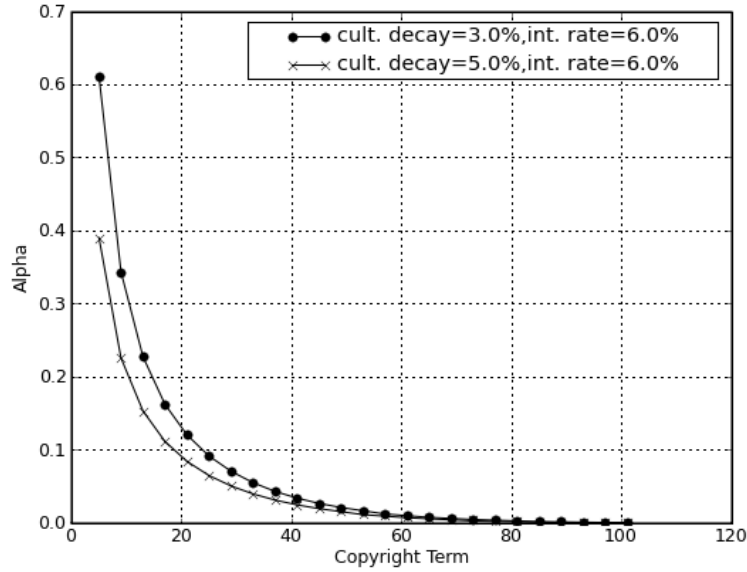


FIGURE 4. Break-even α as a function of copyright term. b is the cultural decay factor and d the discount factor.

Given our assumption on the form of the discount factor and the rate of cultural decay, we have the following expression for the ‘break-even’ deadweight loss ratio:

$$\alpha^* = \alpha^{-1}(S) = \frac{d^S B(\infty)}{\frac{DB(S)}{s^\gamma} + d^S B(S)}$$

Figure 4 provides a plot of this inverse, ‘break-even’, function. Under the Berne convention minimal terms of protection for most types of work is life plus 50 years (and many countries including the US and all of those in the EU now provide for life plus 70). This in turn will correspond to a copyright length of somewhere between 70 and 120 years (assuming the work is created between 0 and 50 years before the creator’s death). Let us take a low value in this range, say 80 years. We summarize the ‘break-even’ α corresponding to term of this length in Table 2 focusing on a set of very conservative parameter values. As can be seen there, even with a cultural decay rate of 2%, a discount rate of 4% and elasticity at its uppermost value the break-even α is 2.4% – that is if deadweight loss is more than 2.4% of total surplus available on a work then current term is too long. With a slightly higher decay and discount rate (3% and 5% respectively) break-even α falls to 1%. Thus, even with low values for the discount and cultural decay rate the level of deadweight loss required for current copyright terms to be optimal seem too low to be plausible.

Cultural Decay Rate (%)	Interest Rate (%)	Elasticity	Break-even α (%)
2	4	1.5	2.4
2	4	1.0	1.6
3	5	1.5	1.0
3	5	1.0	0.7

TABLE 2. Break-even α (per work ratio of deadweight loss to available surplus). γ is set to 0.129.

6. CONCLUSION

In this paper we have developed a simple dynamic model for analysing copyright term. In Proposition 1 we derived a single, simple, equation that defined optimal term as a function of key exogenous variables. Using the estimates for these variables derived from the available empirical data we obtained a point estimate for optimal copyright term of approximately 15 years (with a 99% confidence interval extending up to 38 years). To our knowledge this is the first such estimate which is properly grounded, both theoretically and empirically.

This result has significant implications for policy. Copyright term is probably the most important aspect of the overall ‘level’ of copyright. The estimate obtained for optimal term (15 years) is far below the length of copyright in almost all jurisdictions. Furthermore, while an exact point estimate is obviously subject to considerable variation due to the uncertainty in the underlying parameters, we confirmed using a variety of robustness checks that current copyrights are almost certainly too long. This implies that there is a significant role for policymakers to improve social welfare by reducing copyright term as well as indicating that existing terms should not be extended. Such a result is particularly important given the degree of recent debate on this exact topic.

Finally, there remains plentiful scope to extend and build upon the work here. The empirics in particular are necessarily somewhat crude given the data available. The main challenge then is to improve the estimates for the key parameters, especially that of the ratio of deadweight loss to available surplus. As discussed above, the perfect approach would involve estimating the demand-system for the copyrightable goods under consideration. This is a non-trivial task but one of great value – and with implications for areas other than that considered here.

APPENDIX A. PROOF OF PROPOSITION 1

As we are going to take derivatives we shall take all necessary variables (number of works, time etc) to be continuous rather than discrete (and therefore sums over works should become integrals etc). Note that the conversion back to the discrete version is straightforward (but would make the notation and proof substantially more cumbersome).

Recall from equation 3.10 that we can express steady-state welfare per period as:

$$\begin{aligned} W &= (\text{Potential}) \text{ welfare (all works)} - \text{Deadweight loss on works in copyright} \\ &= Y(n)B(\infty) - Z(n)B(S) \end{aligned}$$

The first term is the welfare available, while the second term subtracts out the deadweight loss for those in copyright. Differentiating:

$$\begin{aligned} \frac{dW}{dS} &= n' \left(y(n)B(\infty) - Z'(n)B(S) \right) - b(S)Z(n) \\ &= \text{Gain in welfare from new works} - \text{Extra deadweight loss on existing works} \end{aligned} \tag{A.1}$$

Let us re-express the increase in the number of works, $n'(S^1)$, in terms of the change in revenue R . As discussed at some length above, formally there is no single revenue variable R . Rather there is a whole set of (potential) works each with an associated revenue and cost with revenues in turn a function of the level of existing and future production and the term of copyright. Production, n is then the number of works such that the marginal producer just breaks even, and is therefore a function of the whole set of revenues and costs. However here matters are simplified for two reasons. First, in calculating marginal changes in n one need only be concerned with changes in the revenue and costs of marginal (potential) works (works already profitable remain profitable when term is increased and works far from being profitable also remain far from being profitable). Second, the underlying change is a change in copyright term. This will only effect revenues leaving costs unchanged.³³ Furthermore, using the expression for revenue in equation (3.13) revenue under a term of S is:

$$R_j(S) = \sum_{t=0}^{t=S} d(t)b(t)r_j(0) = r_j(0)DB(S)$$

³³Recall that the effects of copyright on reuse are being ignored in this paper.

Hence the marginal change in revenue with respect to a change in term is $R'_j(S) = r_j(0)d(S)b(S)$ and the percentage change in revenue, R'_j/R_j is independent of j .

$$\frac{R'_j}{R_j} = \frac{d(S)b(S)}{DB(S)}$$

Returning, then, to an expression for n' . Let k_n be the index of the marginal work, R_{k_n} the revenue, then:

$$n' = \frac{dn}{dS} = \frac{n}{n} \frac{dn(R_{k_n}, C_n)}{dS} = n \frac{dn}{dR_{k_n}} \frac{R_{k_n}}{n} \frac{R'}{R}$$

The middle term of the final expression is the elasticity of supply with respect to revenue (or the marginal work), $s(n)$, while the last is the percentage increase in revenue. Substituting gives:

$$n' = ns(n) \frac{d(S)b(S)}{DB(S)}$$

Finally, substituting this into the (A.1) we have that marginal welfare is:

$$\begin{aligned} \frac{dW(S^1)}{dS} &= b(S^1) \left(\frac{s(n)d(S^1)B(\infty)y(n)}{DB(S^1)} - \frac{s(n)d(S^1)B(S^1)z(n)}{DB(S^1)} - Z(n) \right) \\ &= ns(n)y(n)b(S^1) \left(\frac{d(S^1)}{DB(S^1)} \left(B(\infty) - B(S^1) \frac{z(n)}{y(n)} \right) - \frac{Z(n)}{ns(n)y(n)} \right) \\ &= ns(n)y(n)b(S^1) \cdot \Delta \end{aligned}$$

As the other terms apart from Δ are positive, $\frac{dW}{dS} = 0$ if and only if $\Delta = 0$. Thus, if the optimal copyright term is interior (i.e. non-zero and non-infinite) it must be a solution of $\Delta = 0$. Our next step then is to determine whether a solution exists to $\Delta = 0$ and, if so, whether it is unique.

A.0.1. *Existence and Uniqueness.* One would always assume that $\lim_{S \rightarrow 0} W' > 0$ (with a zero copyright term increasing term is welfare improving). It should also be clear that $\lim_{S \rightarrow \infty} W' = 0$ (Δ is bounded and the term outside the bracket tends to zero).³⁴ Now focus on $\frac{Z(n)}{ns(n)y(n)} = \theta(n)$. This is the ratio of *average* deadweight loss to welfare from a new work. Allowing that the ratio of deadweight loss to welfare from a work ($z(n)/y(n)$) is bounded below and that $s(n)$ is bounded above one has that $\theta(n)$ is bounded below.³⁵ Then for any reasonable discount factor, in particular one which has $\lim_{S \rightarrow \infty} d(S) = 0$, one must have: $\lim_{S \rightarrow \infty} \Delta < 0$. This implies that $\lim_{S \rightarrow \infty} W' = 0^-$.

³⁴Note that this shows that one cannot have the simple situation that $W'' < 0$.

³⁵Let α be minimal ratio of z/y . Then $Z(n)/ny(n) \geq \alpha$.

Combined with $\lim_{S \rightarrow 0} W' > 0$ this implies, by the intermediate value theorem that there exists at least one local maximum S^o at which $\Delta = 0$ and $W'' < 0$. It also shows that $S = \infty$ is not welfare-maximizing and hence that optimal copyright term is ‘interior’ and hence a solution of $\Delta = 0$.

For uniqueness one obviously requires that there is only one solution to $\Delta(S) = 0$. Within the general framework above it is not possible to show this. For example, by choosing an ‘unusual’ elasticity of supply, $s(n)$, in which elasticity suddenly increases sharply at several points one can construct examples in which there are multiple (local) maxima. However, such cases are fairly pathological. In particular the following (together) provide sufficient, though by no means necessary, conditions for Δ to be decreasing and therefore have a unique solution:³⁶

- (1) $\frac{d(S)}{DB(S)}$ is non-increasing. This condition must hold in all reasonable cases – recall that $d(t)$ is the discount factor to time t .
- (2) $B(S) \frac{z(n)}{y(n)}$ is non-decreasing. Clearly $B(S)$ is increasing in S . The second term is the ratio of dead-weight loss to welfare from the n th work which could reasonably be assumed to be constant or increasing.
- (3) $\frac{Z(n)}{ns(n)y(n)} = \theta(n)$ is non-decreasing (at least beyond the first zero of Δ). This is a fairly weak condition. θ is the ratio of *average* deadweight loss to welfare from a new work. With diminishing returns for new work this is likely to be non-decreasing.³⁷

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³⁶Strictly this only shows that it is non-increasing. However by requiring one of these inequalities to be strict the result follows.

³⁷The only surprise would be some kink in the supply curve so that at some point there was a sudden jump in the elasticity of supply $s(n)$.

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